

4th Annual Mathematics Olympiad

GROUP COMPETITION

Barry University

November 9, 2018

School Name: _____

Mutual Non-Attack: Rooks

Two-dimensional board:

A rook attacks other pieces on a chessboard by sliding along vertical or horizontal lines of squares and taking the place of any opponent it encounters.

- a) What is the maximal number of rooks that can be placed on a $n \times n$ chessboard so that no rook is in a position to attack any other rook (a configuration of mutual non-attack)?
- b) How many different ways are there to place a maximal number of rooks on a $n \times n$ board in a mutual non-attack?



Three-dimensional lattice:

Imagine a rook standing within one cell of a cubical lattice. Assume it can move in straight lines up, down, left, right, back or forth to attack other pieces.

- c) In a $3 \times 3 \times 3$ cubical lattice (see figure), what is the maximal number of rooks that can be placed in a configuration of mutual non-attack?
- d) In how many ways can they be placed?
- e) Can you find the maximal number of rooks that can be placed in a configuration of mutual non-attack for $n \times n \times n$ cubical lattice?

Please justify your answer!

Solutions: a) n , b) $n!$ c) 3^2 d) 12 e) n^2